NAG Toolbox for MATLAB

g01gb

1 Purpose

g01gb returns the lower tail probability for the noncentral Student's t-distribution, via the function name.

2 Syntax

```
[result, ifail] = g01gb(t, df, delta, tol, 'maxit', maxit)
```

3 Description

The lower tail probability of the noncentral Student's t-distribution with ν degrees of freedom and noncentrality parameter δ, $P(\mathbf{t} \leq t : \nu;\delta)$, is defined by

$$P(T \leq t : \nu;\delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2}\, dx \right) u^{\nu-1} e^{-u^2/2} du, \qquad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma\left(\frac{1}{2}\nu\right) 2^{(\nu-2)/2}}, \qquad \alpha = \frac{t}{\sqrt{\nu}}.$$

The probability is computed in one of two ways.

(i) When $t = 0.0$, the relationship to the normal is used:

$$P(T \leq t : \nu;\delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2}\, du.$$

(ii) Otherwise the series expansion described in Equation 9 of Amos 1964 is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 References

Amos D E 1964 Representations of the central and non-central t-distributions *Biometrika* **51** 451–458

5 Parameters

5.1 Compulsory Input Parameters

1: **t – double scalar**

t, the deviate from the Student's t-distribution with ν degrees of freedom.

2: **df – double scalar**

ν, the degrees of freedom of the Student's t-distribution.

Constraint: **df** ≥ 1.0.

3: **delta – double scalar**

δ, the noncentrality parameter of the Students t-distribution.

4: **tol – double scalar**

The absolute accuracy required by you in the results. If g01gb is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see x02aj), then the value of $10 \times$ ***machine precision*** is used instead.

5.2 Optional Input Parameters

1: **maxit – int32 scalar**

The maximum number of terms that are used in each of the summations.

Suggested value: 100. See Section 8 for further comments.

Default: 100

Constraint: **maxit** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

If on exit **ifail** $= 0$, then g01gb returns 0.0.

ifail $= 1$

On entry, **df** < 1.0.

ifail $= 2$

On entry, **maxit** < 1.

ifail $= 3$

One of the series has failed to converge. Reconsider the requested tolerance and/or maximum number of iterations.

ifail $= 4$

The probability is too small to calculate accurately.

7 Accuracy

The series described in Amos 1964 are summed until an estimated upper bound on the contribution of future terms to the probability is less than **tol**. There may also be some loss of accuracy due to calculation of gamma functions.

8 Further Comments

The rate of convergence of the series depends, in part, on the quantity $t^2/\left(t^2+\nu\right)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the t-distribution to the F-distribution can be used:

$$F = T^2, \lambda = \delta^2, \nu_1 = 1 \qquad \text{and} \qquad \nu_2 = \nu,$$

and a call made to g01gd.

Note that g01gb only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

9 Example

```
t = -1.528;
df = 20;
delta = 2;
[result, ifail] = g01gb(t, df, delta)
```

```
result =
   3.1800e-04
ifail =
           0
```